RADIONUCLEID PROPAGATION IN A POROUS MEDIUM WITH ALLOWANCE MADE FOR SORPTION AND DIFFUSION

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An analytic solution is found for the concentration of the first and the second radionucleids in a porous medium. The effect is shown of radioactive decay and of diffusion on the occurrence time of the maximal concentration at the outlet of the absorber layer.

In [1] equilibrium adsorption was studied of a radioactive admixture which is carried by a carrier gas with constant rate through a semiinfinite porous medium. In the present article the effect of diffusion is taken into account on the motion of an isobar collection of radioactive nucleids in a porous medium.

The transport of an isobar collection of radionucleids by means of a nonactive carrier gas is considered. The concentration propagation of the admixture in the flow is described by a system of parabolic-type equations:

$$\frac{\partial c_1}{\partial t}+\frac{v\gamma}{1+\gamma}\frac{\partial c_1}{\partial x}+\lambda_1 c_1-\frac{D\gamma}{1+\gamma}\frac{\partial^2 c_1}{\partial x^2}=0, \tag{1}$$

$$\frac{\partial c_i}{\partial t}+\frac{v\gamma}{1+\gamma}\frac{\partial c_i}{\partial x}+\lambda_i c_i-\frac{D\gamma}{1+\gamma}\frac{\partial^2 c_i}{\partial x^2}=\lambda_{i-1}c_{i-1} \tag{2}$$

$$(\lambda_N=0,\ i=2,\ \ldots,\ N).$$

The system of equations (1)-(2) was written under the assumption that the characteristic time of establishing the equilibrium between separate radionucleids in the flow and in the adsorbent is much shorter than the characteristic time of the process, that is, the character of adsorption is of equilibrium type. The relation between the concentrations of the i-th nucleid in the flow and the adsorbent is $c_i = \gamma a_i$. Here $i = 1, \ldots, N$. Assuming that at the initial moment there is no radioactive gas in the porous medium, the initial and boundary conditions for the system (1)-(2) are

$$c_1(x,\ 0)=0,\ c_i(x,\ 0)=0,\ c_1(0,\ t)=\mu(t),\ c_i(0,\ t)=\mu_i(t), \tag{3}$$

where $\mu_i(t)$ ($i = 2, \ldots, N$) can be found by solving the following system of equations:

$$\frac{\partial c_i}{\partial t}+\lambda_i c_i-\lambda_{i-1}c_{i-1}=0\quad (i=2,\ \ldots,\ N;\ \lambda_N=0). \tag{4}$$

In practice, a concentration of the radioactive gas is fed most often to the input of the sorbing layer either in the form of an abrupt impulse or a step. We shall consider the case of an abrupt impulse which corresponds to the boundary condition containing a delta-function:

$$\mu_1(t)=\delta(t,\ t_0). \tag{5}$$

Since the impulse of a radioactive gas does not vanish at the boundary, therefore $c_i(0, t) = 0$. The solution for the first radionucleid using (5) is given by

$$c_1(x,\ t)=\frac{x(1+\gamma)^{1/2}}{2(\pi D\gamma)^{1/2}t^{3/2}}\exp\left\{-\frac{[x(1+\gamma)-v\gamma t]^2}{4D\gamma(1+\gamma)t}-\lambda_1 t\right\}. \tag{6}$$

Using the general theory of solutions of parabolic equations [2] it is not difficult to see that the solution $c_i(x, t)$ ($i > 1$) for a vanishing initial and boundary condition can be written as

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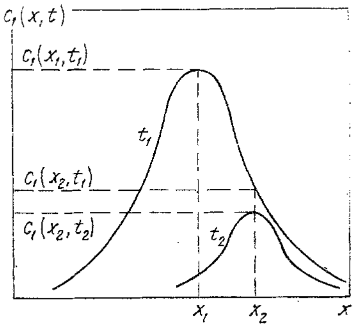

Fig. 1. Concentration of the first radionucleid at the instants t_1 and t_2 versus x (x_1, coordinate of the concentration maximum of the radionucleid at the instant t_1; x_2, coordinate of the concentration maximum of the second radionucleid at the instant t_2).

$$c_i(x,\ t) = \frac{\lambda_{i-1}(1+\gamma)^{1/2}}{2\sqrt{\pi D\gamma}} \exp\left\{\frac{xv}{2D} - \left[\lambda_i + \frac{v\gamma}{4D(1+\gamma)}\right]t\right\}$$
$$\times \int_0^\infty \int_0^t \frac{c_{i-1}(\zeta,\ \tau)}{\sqrt{t-\tau}} \exp\left\{\left[\lambda_i + \frac{v\gamma}{4D(1+\gamma)}\right]\tau - \frac{v\zeta}{2D}\right\}$$
$$\times \left[\exp\left\{-\frac{(x-\zeta)^2(1+\gamma)}{4D\gamma(t-\tau)}\right\} - \exp\left\{-\frac{(x+\zeta)^2(1+\gamma)}{4D\gamma(t-\tau)}\right\}\right] d\zeta d\tau. \tag{7}$$

By substituting in the expression (7) instead of $c_{i-1}(\zeta,\ \tau)$ the known concentration for the first radionucleid, one obtains the concentration of the second radionucleid in the case of motion in a porous medium which contains an integral:

$$c_2(x,\ t) = \frac{\lambda_1 x\sqrt{1+\gamma}}{2t\sqrt{\pi D\gamma t}} \exp\left\{-\frac{x^2(1+\gamma)}{4D\gamma t} + \frac{xv}{2D} - \left[\lambda_2 + \frac{v\gamma}{4D(1+\gamma)}\right]t\right\}$$
$$\times \int_0^t \operatorname{erf}\left[\frac{x}{2}\sqrt{\frac{\tau(1+\gamma)}{D\gamma t(t-\tau)}}\right] \exp\{-(\lambda_1-\lambda_2)\tau\}\, d\tau. \tag{8}$$

In the above $\operatorname{erf}(z) = \frac{1}{\sqrt{2\pi}}\int_{-z}^{z} \exp\left(-\frac{y^2}{2}\right) dy$ is the error integral. An exact calculation of (8) is not feasible. By employing an asymptotic representation of the error integral [3], the concentration of the second radionucleid can be written approximately as follows:

$$c_2(x,\ t) \simeq \frac{x\sqrt{t+\gamma}\,\lambda_1}{2t\sqrt{\pi D\gamma t}\,(\lambda_1-\lambda_2)} \exp\left\{-\frac{x^2(1+\gamma)}{4D\gamma t} + \frac{xv}{2D} - \frac{v\gamma t}{4D(1+\gamma)}\right\}[\exp(-\lambda_2 t) - \exp(-\lambda_1 t)]. \tag{9}$$

It is noted that it is evident from (9) that the approximate solution for the second radionucleid was written in the form of a product of two factors, one of which characterizes only the decay, the other being a solution of the equation which describes the transport of the admixture with diffusion and sorption taken into account. One would expect that the approximate solutions for the third and the subsequent radionucleids would look similar to (9). However, it is inadvisable to employ the expression (7) to obtain analytically the concentrations of the radionucleids starting from the third one upward though it is suitable for computing the concentrations of the set of radionucleids on a high-speed electronic computer.

It is not difficult to see from the expression (6) that the coordinate of the concentration maximum in the flow is given by the following expression:

$$x = \frac{v\gamma t}{2(1+\gamma)}\left[1 + \sqrt{1 + \frac{2D(1+\gamma)}{v^2\gamma t}}\right]. \tag{10}$$

For $t \gg D(1+\gamma)\gamma^{-1}\nu^{-2}$ the expression (10) can be simplified to:

$$x = \frac{v\gamma t}{1+\gamma}. \tag{11}$$

As can easily be seen from (11) the rate of motion of the concentration maximum in the medium for large t only depends on the Henri coefficient $x_t = \nu\gamma(1+\gamma)^{-1}$. It is noted that the arrival time of the concentration maximum at the outlet of the adsorber can be determined from Eq. (10):

$$t = \frac{x(1+\gamma)}{\gamma\nu}\left(1+\frac{2D}{x\nu}\right)^{-1}. \tag{12}$$

If $x\nu \gg D$, then (12) can be simplified as well:

$$t = \frac{x(1+\gamma)}{\nu\gamma}. \tag{13}$$

However, the appearance time of the concentration maximum for a radioactive admixture at the absorber outlet differs from that calculated from (12) and (13); the former can be found from an equation obtained by differentiating (6) with respect to t:

$$t = \frac{x(1+\gamma)}{\nu\sqrt{\gamma^2+4D\gamma\lambda_1\nu^{-2}(1+\gamma)}}\sqrt{1+\frac{9D^2\gamma}{x^2\nu^2[\gamma+4D\lambda_1\nu^{-2}(1+\gamma)]}} - \frac{3D(1+\gamma)}{\nu^2[\gamma+4D\nu^{-2}\lambda_1(1+\gamma)]}. \tag{14}$$

For $\nu x \gg D$ one can simplify (14) as well:

$$t = \frac{x(1+\gamma)}{\nu\gamma\sqrt{1+4D\nu^{-2}\lambda_1\gamma^{-1}(1+\gamma)}} \tag{15}$$

Comparing (13) and (15), one can see that for $D = 0$ or for $\lambda_1 = 0$ (the gas being inactive or the diffusion coefficient being negligibly small) the expressions (13) and (15) are identical. The effect of different appearance times for the concentration maximum of an active or inactive admixture at the adsorber outlet is due to the fact that during the motion of the active admixture its concentration reduces owing to radioactive decay, and when the "proper" maximum arrives at the outlet of the sorbing layer its amount will be smaller than the concentration at the adsorber outlet at some previous instant t. Figure 1 shows the discrepancy between the appearance times of the concentration maxima for an active or an inactive admixture at a point x_2. In Fig. 1 the concentration $c_1(x,t)$ is shown of the first radionucleid versus x at the time instants t_1 and t_2 ($t_1 < t_2$). It can be seen that though the concentration maximum at the instant t_1 is at the point x_1, one nevertheless has $c_1(x_2, t_1) < c_1(x_2, t_2)$. It is evident that the appearance time of the concentration maximum at the adsorber outlet is the same also for an active or inactive admixture provided the diffusion coefficient vanishes. The latter, however, is of little importance and is due to the specific features of our problem. Since it is assumed that the concentration is transported to the adsorber inlet in the delta-function form, with no diffusion over the medium the delta-impulse spreads; since it is not washed away, there are no differences in the appearance times of the maxima for an active or inactive admixture.

In practical applications, the Henri coefficient for an inactive admixture is determined if the motion velocity of the basic carrier gas ν is known as well as the occurrence time T of the concentration maximum at the adsorber outlet and the adsorber length l according to the formula

$$\gamma = \frac{l}{\nu T - l}. \tag{16}$$

With an additional assumption $\gamma \gg D\lambda(\nu^2 - D\lambda)^{-1}$ it is not difficult to find from (15) an expression for the Henri coefficient of an active admixture:

$$\gamma = \frac{l - 2D\lambda_1\nu^{-1}T}{\nu T - l + 2D\lambda_1\nu^{-1}T}. \tag{17}$$

It is clear from (17) that to determine the Henri coefficient it is not enough to know the velocity of motion for the basic gas, the length of the adsorber, and the appearance instant of the concentration maximum at the outlet from the adsorber layer, as was the case with the inactive admixture, but one must also know the diffusion coefficient and the radioactive decay constant.

It is noted that if the coefficient of lengthwise diffusion in the medium is unknown, it can be found by passing an active and inactive gas through a porous medium and by measuring the appearance times of the concentration maxima of the active (T_1) or inactive admixture (T):

$$D = \frac{\nu(T^2 - T_1^2)}{4\lambda_1 T_1^2 T}. \tag{18}$$

Similarly to the expressions (11) and (15), one can find from (18) the coordinate of the concentration maximum of the second radionucleid of the set as well as the appearance time of the concentration maximum at the adsorber outlet.

One should mention, in conclusion, that the results obtained in this article can be employed in the analysis of radionucleid migration in the earth and also to investigate radioactive gases by chromatography.

NOTATION

$c_i(x, t)$, concentration of the i-th radionucleid in flow; $a_i(x, t)$, concentration of the i-th nucleid in adsorbent; γ, reciprocal of Henri coefficient; λ_i, decay constant of the i-th radionucleid; D, diffusion coefficient; ν, velocity of the main gas carrier.

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ACOUSTIC METHOD OF INVESTIGATING NONSTATIONARY HEAT CONVECTION IN CYLINDRICAL LAYERS OF GASES AND LIQUIDS

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A new acoustic method of investigating nonstationary heat transfer and heat conduction in gases and liquids is described. The apparatus is described and experimental results for air are given.

The hot-wire method or, as it is sometimes called, the cylindrical probe of constant power method [1-5, 14], is widely used to study heat transfer in liquids and gases. Its advantage is the relative simplicity of the measuring cell. This method has also been used to investigate heat conduction and heat convection under steady conditions. The main features of nonstationary free convection have not been studied to any great extent [6-9, 15, 16].

In this paper we use the hot-wire nonstationary acoustic method to investigate the heat-transfer properties of liquids and gases. The method is based on measuring the phase difference or frequency difference (for high heating speeds) of ultrasonic oscillations in fine wires [10-11]. The same wire serves both as a heater and for measuring the temperature.

The response time of the measuring probe, determined by the time taken for the acoustic signal to propagate through the control part of the medium, is of the order of $5 \cdot 10^{-5}$ sec. Hence, a measurement can be made immediately after connecting or disconnecting the source of heat, the time taken to carry out the experiments thereby being reduced to several seconds. The method enables one to study the development of convective heat transfer at high heating or cooling rates, since the resolving power of the frequency method increases when the rate of variation of the temperature is increased (the nonstationary mode), while the relative error in measuring the frequency shift is reduced. Among the features of the method is the fact that information on the

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